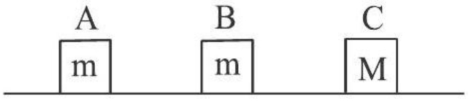
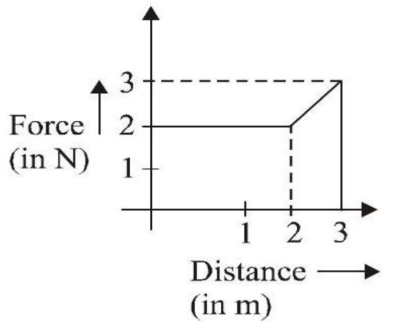
Work, Energy and Power

1. There block A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically and $\frac{5}{6}$ th of the initial kinetic energy is lost in whole process. If the value of M/m is $\frac{4}{x}$. Find the value of x.



2. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done (in joule) by this force in first 5 seconds?
3. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity $100\ ms^{-1}$, from the ground. The bullet gets embedded in the wood. Then the maximum height (in metre) to which the combined system reaches above the top of the building before falling below is: $(g = 10\ ms^{-2})$
4. A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\hat{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy (in joule) at the end of the displacement?
5. A body of mass 1 kg falls freely from a height of 100 m, on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6\ N/m$. The body sticks to the platform and the spring's maximum compression is found to be x cm. Given that $g = 10\ ms^{-2}$, the value of x will be close to :
6. An alpha-particle of mass m suffers one-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is x m. Find the value of x.
7. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy (in joule) of the particle after it has travelled 3 m is :



8. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass (in kg) of the second body?
9. A body of mass 3 kg is under a constant force which causes a displacement s in metre in it, given by the relation $s = \frac{1}{3}t^3$, where t is in second. Work done (in joule) by the force in 2 second is

10. A ball is dropped from a height of 100 m. After striking the earth, 20% of its energy is lost. To what height (in metre) the ball will rise?

11. A force of 5 N making an angle θ with the horizontal, acting on an object displaces it by 0.4 m along the horizontal direction. If the object gains kinetic energy of 1 J, the horizontal component of the force (in newton) is

12. A motor of 100 H.P. moves a load with a uniform speed of 72 $\mathrm{km/h}$. The forward thrust (in newton) applied by the engine of the car is

13. Four smooth steel balls of equal mass at rest are free to move along a straight line without friction. The first ball is given a velocity of 0.4 $\mathrm{m/s}$. It collides head on with the second elastically, the second one similarly with the third and so on. The velocity (in $\mathrm{m/s}$) of the last ball is

14. A shell of mass 20 kg at rest explodes into two fragments whose masses are in the ratio $2:3$. The smaller fragment moves with a velocity of 6 $\mathrm{m/s}$. The kinetic energy (in joule) of the larger fragment is

15. A ball drops from a celling of a room and after rebounding twice from the floor reaches a height equal to half that of the celling. If the coefficient of restitution is $\left(\frac{1}{2}\right)^{1/x}$. Find the value of x.

SOLUTIONS

1. **(1)** Kinetic energy of block A

$k_1 = \frac{1}{2}mv_0^2$

$\therefore$ From principle of linear momentum conservation

$mv_0 = (2m+M)v_f$ Þ $v_f = \frac{mv_0}{2m+M}$

According to question, of $\frac{5}{6}$th the initial kinetic energy is lost in whole process.

$\therefore \frac{k_i}{k_f} = 6 \Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m+M)\left(\frac{mv_0}{2m+M}\right)^2} = 6$

Þ $\frac{2m+M}{m} = 6 \therefore \frac{M}{m} = 4$

2. **(900)** Position, $x = 3t^2 + 5$

$\therefore$ Velocity, $v = \frac{dx}{dt}$

$\Rightarrow v = \frac{d(3t^2+5)}{dt}$

$\Rightarrow v = 6t + 0$

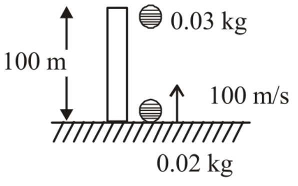
At $t = 0$ $\quad v = 0$

And, at $t = 5$ sec $\quad v = 30$ m/s

According to work-energy theorem, $w = \Delta KE$

or, $W = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(2)(30)^2 = 900J$

3. **(40)**



Time taken for the particles to collide,

$t = \frac{d}{v_{rel}} = \frac{100}{100} = 1s$

Speed of wood just before collision = gt = 10m/s and speed of bullet just before collision = v – gt = 100 – 10 = 90 m/s

$S = 100 \times 1 - \frac{1}{2} \times 10 \times 1 = 95$ m

Now, using conservation of linear momentum just before and after the collision

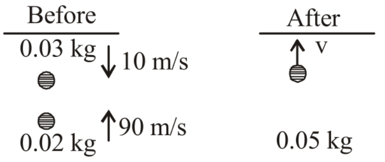
$-(0.03)(10) + (0.02)(90) = (0.05)v$

$\Rightarrow 150 = 5v$

$\therefore \; v = 30$ m/s

Max. height reached by body

$$h = \frac{v^2}{2g} = \frac{30\times30}{2\times10} = 45\,m$$



$\therefore$ Height above tower = 40 m

4. **(15)** Work done = $\vec{F}.\vec{d} = \left(3\vec{i} - 12\vec{j}\right).\left(4\vec{i}\right) = 12J$

From work energy theorem,

$w_{net} = \Delta K.E. = k_f - k_i$

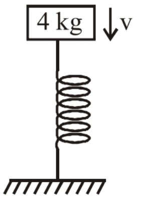
$\Rightarrow 12 = k_f - 3$

$\therefore K_f = 15J$

5. **(4)** Velocity of 1 kg block just before it collides with 3 kg block $= \sqrt{2gh} = \sqrt{2000}$ m/s

Using principle of conservation of linear momentum just before and just after collision, we get

$$1\times\sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4}\ \text{m/s}$$



initial compression of spring

$1.25\times10^6 x_0 = 30 \Rightarrow x_0 \approx 0$

using work energy theorem,

$W_g + W_{sp} = \Delta KE$

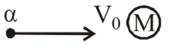
$$\Rightarrow 40\times x + \frac{1}{2}\times1.25\times10^6(0^2 - x^2)$$

$$= 0 - \frac{1}{2}\times4\times v^2$$

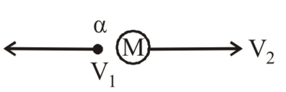
solving $x \approx 4$ cm

6. **(4)** Using conservation of momentum,

$mv_0 = mv_2 - mv_1$



After collision



$$\frac{1}{2}mv_1^2 = 0.36\times\frac{1}{2}mv_0^2$$

$$\Rightarrow v_1 = 0.6v_0$$

The collision is elastic. So,

$$\frac{1}{2}MV_2^2 = 0.64\times\frac{1}{2}mv_0^2\ [\because M = \text{mass of nucleus}]$$

$$\Rightarrow V_2 = \sqrt{\frac{m}{M}}\times0.8V_0$$

$$mV_0 = \sqrt{mM}\times0.8V_0 - m\times0.6V_0$$

$$\Rightarrow 1.6m = 0.8\sqrt{mM}$$

$$\Rightarrow 4m^2 = mM$$

$$\therefore M = 4m$$

7. **(6.5)** We know area under F-x graph gives the work done by the body

$$\therefore W = \frac{1}{2}\times(3+2)\times(3-2)+2\times 2$$

$$= 2.5 + 4$$

$$= 6.5 \text{ J}$$

Using work energy theorem,

Δ K.E = work done

$\therefore$ Δ K.E = 6.5 J

8. **(1.2)** For head on elastic collision we have

$$V_1 = \frac{(m_1-m_2)u_1}{m_1+m_2}+\frac{2m_2u_2}{m_1+m_2}$$

Here m_1 = 2kg, u_1 = x, u_2 = 0,

v_1 = x/4

$$\therefore \frac{x}{4}=\frac{(2-m_2)x}{2+m_2} \Rightarrow m_2 = 1.2\text{kg}$$

9. **(24)** Acceleration $= \frac{d^2s}{dt^2} = 2t\ ms^2$

$$w=\int_0^2 ma.ds$$

$$=\int_0^2 3\times 2t\times t^2\, dt$$

Solving we get

$w = 24$J

10. **(80)** $mgh^1 = \frac{80}{100}\times mg\times 100 \Rightarrow h^1 = 80m$

11. **(2.5)** Work done on the body = K. E. gained by the body

$$Fs\cos\theta = 1 \Rightarrow F\cos\theta = \frac{1}{s} = \frac{1}{0.4} = 2.5N$$

12. **(3730)** Forward thrust, $F = \frac{P}{v} = \frac{100\times 746}{20} = 3730\text{ N}.$

13. **(0.4)** In an elastic head-on collision, of two equal masses their kinetic energies or velocities are exchanged. Hence when the first ball collides with the second ball at rest, the second ball attains the speed of 0.4 m/s and the first ball comes to rest. This process continues. Thus the velocity of the last ball is 0.4 ms^{-1}.

14. **(96)**

15. **(4)** $h_n = e^{2n}h_0 \Rightarrow \frac{h}{2} = e^4 h \Rightarrow e = \left(\frac{1}{2}\right)^{1/4}$